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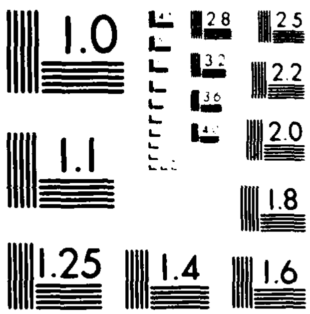

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Technical Report 644

T.S. Seay

Close Approaches Between Satellites in Circular Orbits

25 August 1983

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This technical report has been reviewed and is approved for publication.

FOR THE COMMANDER

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
LINCOLN LABORATORY

CLOSE APPROACHES BETWEEN SATELLITES IN CIRCULAR ORBITS

T.S. SEAY

Group 64

TECHNICAL REPORT 644

25 AUGUST 1983

LEXINGTON MASSACHUSETTS

ABSTRACT

The variation in range between two satellites in different circular orbits is of interest in many space system problems. This report provides a simple expression for the range and presents an analysis of the frequency and duration of close approaches. The results are applied to a simple satellite search problem. Some relationships for the range rate are also given.

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I. INTRODUCTION

The purpose of this report is to describe the behavior of the range between two spacecraft in different circular orbits. Specifically, we have a spacecraft, denoted by subscript 0, in a circular orbit of radius R_0 and corresponding period T_0, which carries a sensor or other system characterized by an effective radius R_S. What is the maximum amount of time which can take place before a spacecraft, denoted by subscript 1, with orbital radius R_1, orbital period T_1, and relative inclination I, will come within the sphere defined by R_S? What is the average amount of time that the spacecraft will spend within the range R_S? What is the maximum rate-of-change of the range between the spacecraft?

In Section II a convenient form for the range between the two spacecraft is derived. Section III defines a close approach, and gives the time interval between two such approaches. We find a characteristic interval within which a close approach will occur, irrespective of the relative inclination between the two orbit planes. Section IV presents a calculation of the average time duration that the two spacecraft stay within a specified range, and it is observed that this time is minimized when the two orbit planes are perpendicular. Section V describes an application of the results to a spaceborne search problem. Finally, Section VI presents a simple upper bound on the maximum range rate between the two spacecraft.

II. RANGE BETWEEN SPACECRAFT

Figure 1 illustrates the geometry of interest. The plane of spacecraft 1 is inclined with respect to the plane of spacecraft 0 by relative inclination angle I. This inclination angle is uniquely defined by requiring that for I = 0, both spacecraft are moving in the same direction.

This section presents a derivation for an expression giving the distance between the two spacecraft. Consider two rectangular coordinate systems with the x-axis in each corresponding to the line of intersection between the planes of the two orbits as shown in Fig. 1. The coordinate system 0 will be in the plane of motion of satellite 0 while coordinate system 1 will be in the plane of motion of satellite system 1. In each of the planes, the corresponding equations of motion of the satellites are

$$X_0 = R_0 \sin\omega_0 t$$

$$Y_0 = R_0 \cos\omega_0 t \qquad \text{(II-1)}$$

$$X_1 = R_1 \sin(\omega_1 t + \phi)$$

$$Y_1 = R_1 \cos(\omega_1 t + \phi) \qquad \text{(II-2)}$$

where

$$\omega_i = \frac{2\pi}{T_i} \qquad \text{(II-3)}$$

is the equivalent angular rate of the i^{th} satellite in its plane of motion. We have chosen t = 0 to be the time that satellite 0 is at the intersection of the planes. ϕ is the angular distance from satellite 1 to the intersection of

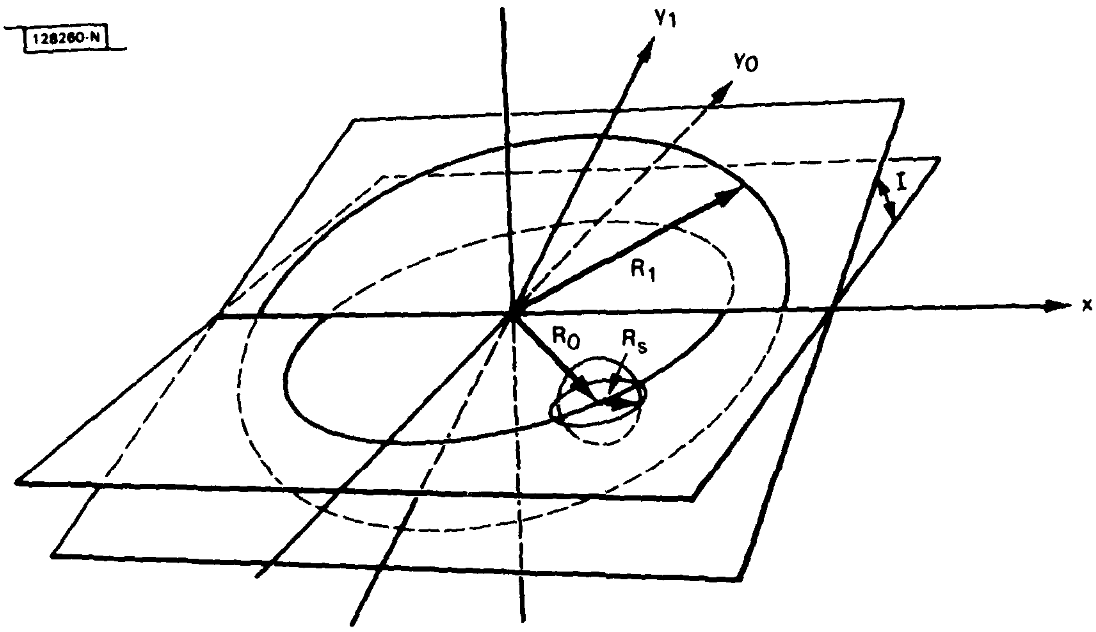

Fig. 1. Problem geometry.

the planes, in plane 1, at $t = 0$. Without loss of generality, we assume $T_1 > T_0$.

Now let X_1', Y_1', Z_1' be the coordinates of satellite 1 expressed in the plane of satellite 0. Then:

$$X_1' = X_1$$

$$Y_1' = Y_1 \cos I$$

$$Z_1' = Y_1 \sin I \qquad . \tag{II-4}$$

Denoting the distance between the two spacecraft as d:

$$d^2 = (X_0 - X_1')^2 + (Y_0 - Y_1')^2 + (0 - Z_1')^2 \quad . \tag{II-5}$$

Substituting, and using

$$1 + \cos X = 2\cos^2 \frac{X}{2} \quad , \tag{II-6}$$

$$1 - \cos X = 2\sin^2 \frac{X}{2} \quad , \tag{II-7}$$

we obtain

$$d^2 = (R_1 - R_0)^2 + 4R_0R_1 \left[\cos^2 \frac{I}{2} \sin^2 \left(\frac{\omega_0 t - \omega_1 t - \phi}{2}\right) + \sin^2 \frac{I}{2} \cos^2 \left(\frac{\omega_0 t + \omega_1 t + \phi}{2}\right)\right] . \tag{II-8}$$

As a simple check, Eq. (II-8) shows that the range varies between $R_0 + R_1$ and $R_0 - R_1$ when I is either 0 or π.

III. CLOSE APPROACHES

For convenience, introduce:

the difference frequency $\omega_\Delta \triangleq \omega_0 - \omega_1$ (III-1)

the sum frequency $\omega_\Sigma \triangleq \omega_0 + \omega_1$ (III-2)

the offset time $\tau \triangleq t - \frac{\phi}{\omega_\Delta}$. (III-3)

In these terms Eq. (II-8) can be rewritten

$$d^2 = (R_1 - R_0)^2 + 4R_0R_1 \left[\cos^2 \frac{I}{2} \sin^2 \frac{\omega_\Delta \tau}{2} + \sin^2 \frac{I}{2} \cos^2 \frac{\omega_\Sigma \tau + \eta}{2} \right] \tag{III-4}$$

where the equivalent angular advance η is:

$$\eta = \phi(1 + \frac{\omega_\Sigma}{\omega_\Delta}) \quad . \tag{III-5}$$

In Eq. (III-4), the [ ] contains a term with the difference frequency period

$$T_\Delta = \frac{T_0 T_1}{T_1 - T_0} \tag{III-6}$$

corresponding to the difference frequency, ω_Δ, and a term with a sum frequency period

$$T_\Sigma = \frac{T_0 T_1}{T_1 + T_0} \tag{III-7}$$

corresponding to ω_Σ.* Qualitatively, d^2 consists of a constant term, a slowly varying positive term with period T_Δ, and a rapidly varying positive term with period T_Σ. (Note that T_Δ is greater than T_0 or T_1, while T_Σ is less than either T_0 or T_1.)

Since both terms within [ ] in Eq. (III-4) are positive, it is clear that the closest possible approach (i.e., $d = R_1 - R_0$) occurs only for those τ such that $\omega_\Delta\tau = k\pi$ for k integral, and $\omega_\Sigma\tau + \eta = (\frac{2\ell+1}{2})\pi$ for ℓ integral. Depending on the particular ω_Δ, ω_Σ and η, the time between such closest possible approaches could be extremely long.

However, as a practical matter, because of the great difference between ω_Σ and ω_Δ for typical cases, we will consider the basic time interval T_Δ. During each (half-open) time interval of duration T_Δ, the term $\sin^2 \frac{\omega_\Delta\tau}{2}$ in Eq. (III-4) has one (and only one) zero. During the same interval, the term $\cos^2 \frac{\omega_\Sigma\tau + \eta}{2}$ will have $\omega_\Sigma/\omega_\Delta$ zeros.** Thus there will be a zero of the $\cos^2$ term occurring within $\frac{1}{2} T_\Sigma$ of the time at which the $\sin^2$ term is zero. Therefore, within every time interval of duration T_Δ, the satellites will come within the distance:

$$d^2_{min} < (R_1 - R_0)^2 + 4R_0R_1 \cos^2 \frac{I}{2} \sin^2 \frac{\pi}{2} \frac{\omega_\Delta}{\omega_\Sigma} \quad . \qquad \text{(III-8)}$$

One implication of this expression is that the relative inclination between the orbital planes has a very small effect upon the range at closest approach for small $\omega_\Delta/\omega_\Sigma$. (Small $\omega_\Delta/\omega_\Sigma$ corresponds to the case where the satellite periods are close to one another.) Furthermore, the characteristic period T_0, within which a close approach is certain to occur, is unaffected by the relative inclination.

---

* From (II-6) and (II-7), the $\sin^2$ and $\cos^2$ terms have equivalent frequencies equal to twice their arguments.

**For example, with 10% difference in periods, the $\cos^2$ term will have 21 zeros in this interval.

IV. DURATION OF CLOSE APPROACHES

In this section the fraction of the basic time interval T_Δ that the distance d is less than a characteristic radius R_S is studied. Specifically, the intent is to determine those time intervals for which

$$R_S^2 \geq (R_1-R_0)^2 + 4R_0R_1 \cos^2 \frac{I}{2} \sin^2 \frac{\omega_\Delta \tau}{2} + \sin^2 \frac{I}{2} \cos^2 \frac{\omega_\Sigma \tau + \eta}{2} \qquad \text{(IV-1)}$$

which is rewritten as

$$\alpha \geq \cos^2 \frac{I}{2} \sin^2 \frac{\omega_\Delta \tau}{2} + \sin^2 \frac{I}{2} \cos^2 \frac{\omega_\Sigma \tau + \eta}{2} \qquad \text{(IV-2)}$$

where

$$\alpha = \frac{R_S{}^2 - (R_1-R_0)^2}{4R_0R_1} \quad . \qquad \text{(IV-3)}$$

Unfortunately, there can be many time intervals within T_Δ which satisfy inequality (IV-2), so that an accurate analytic solution is very difficult. Figure 2 illustrates this problem with a plot of the right-hand side of (IV-2) against the normalized time τ/T_Δ for $I = 0$, $\pi/2$, and π radians. The numerically determined percentage of time that inequality (IV-2) is satisfied (hereafter denoted by f) is plotted in Fig. 3 as a function of I with R_1/R_0 as a parameter for that value of η which minimizes f.* As the reader may expect, $I = \pi/2$ appears to give the lowest f that the two spacecraft are within the specified range. Figure 4 shows f for the case $I = \pi/2$, as a function of R_1/R_o for that value of η minimizing f. The irregularities in these curves appear to be caused by the complicated relationship among ω_Σ, ω_Δ, and α.

Simple analytic upper bounds are obtained by considering each of the two products on the right-hand side of (IV-2) independently. Define the times T_1 and T_2 by the relations:

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* η was tested in steps of $\pi/18$ radians.

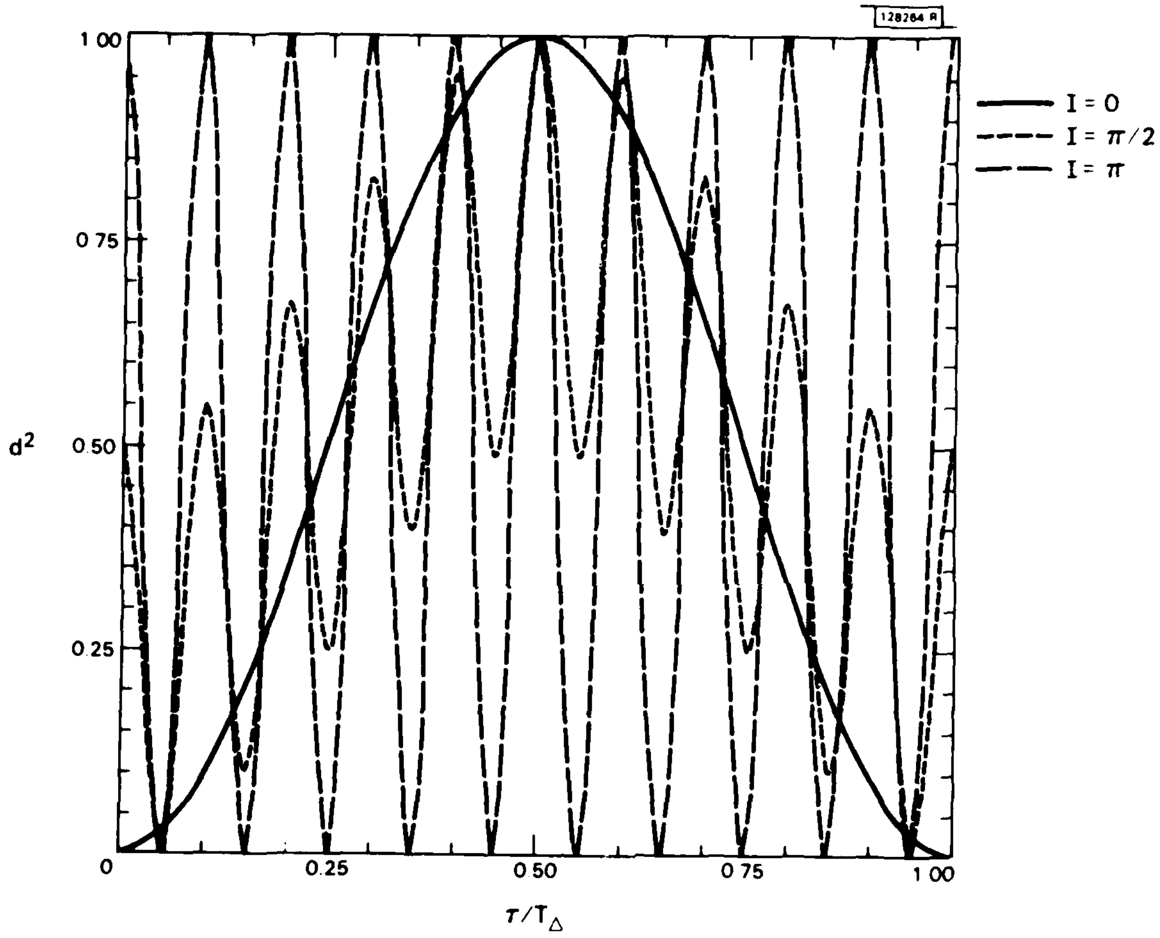

Fig. 2. Normalized squared distance d^2 between two satellites as a function of normalized time τ/T_Δ.

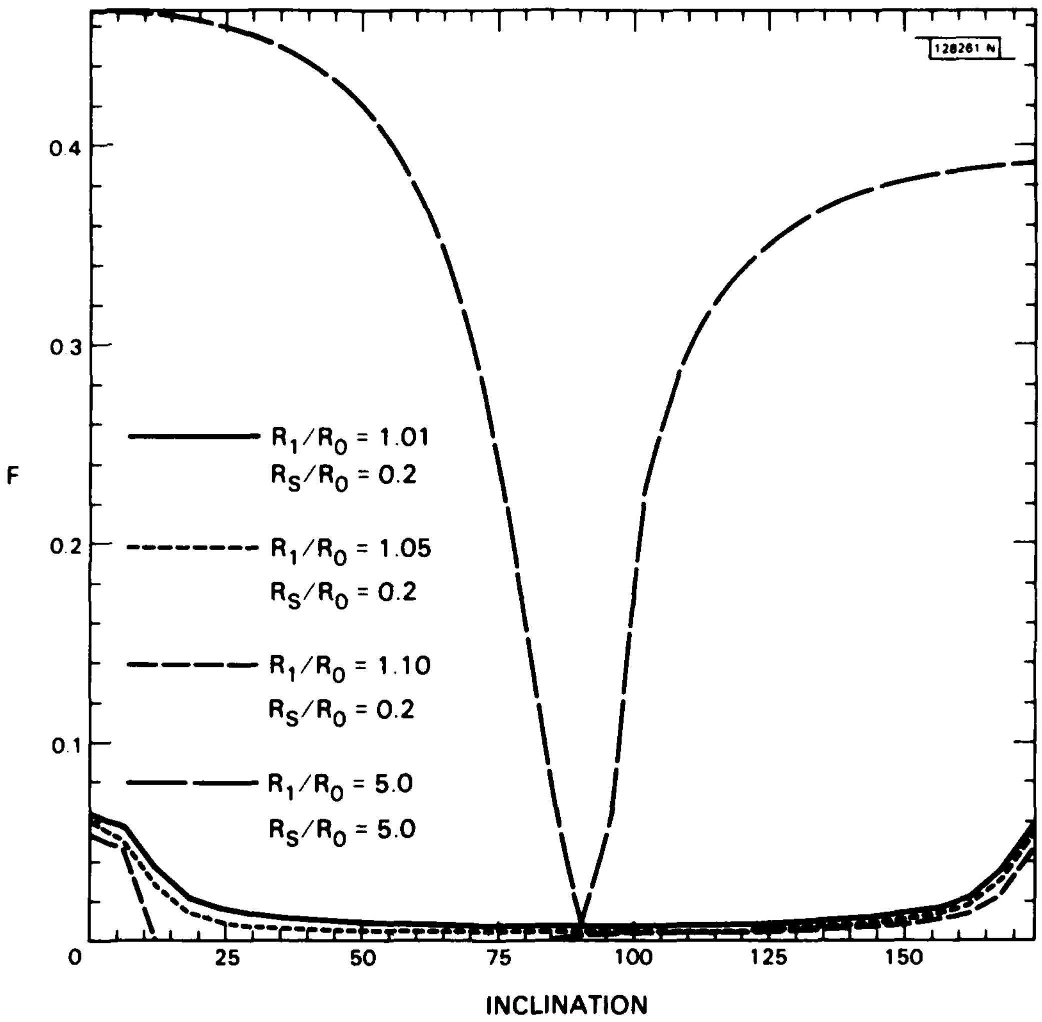

Fig. 3. Fraction of time two satellites are within the squared distance α of one another, for worse case phase angle η.

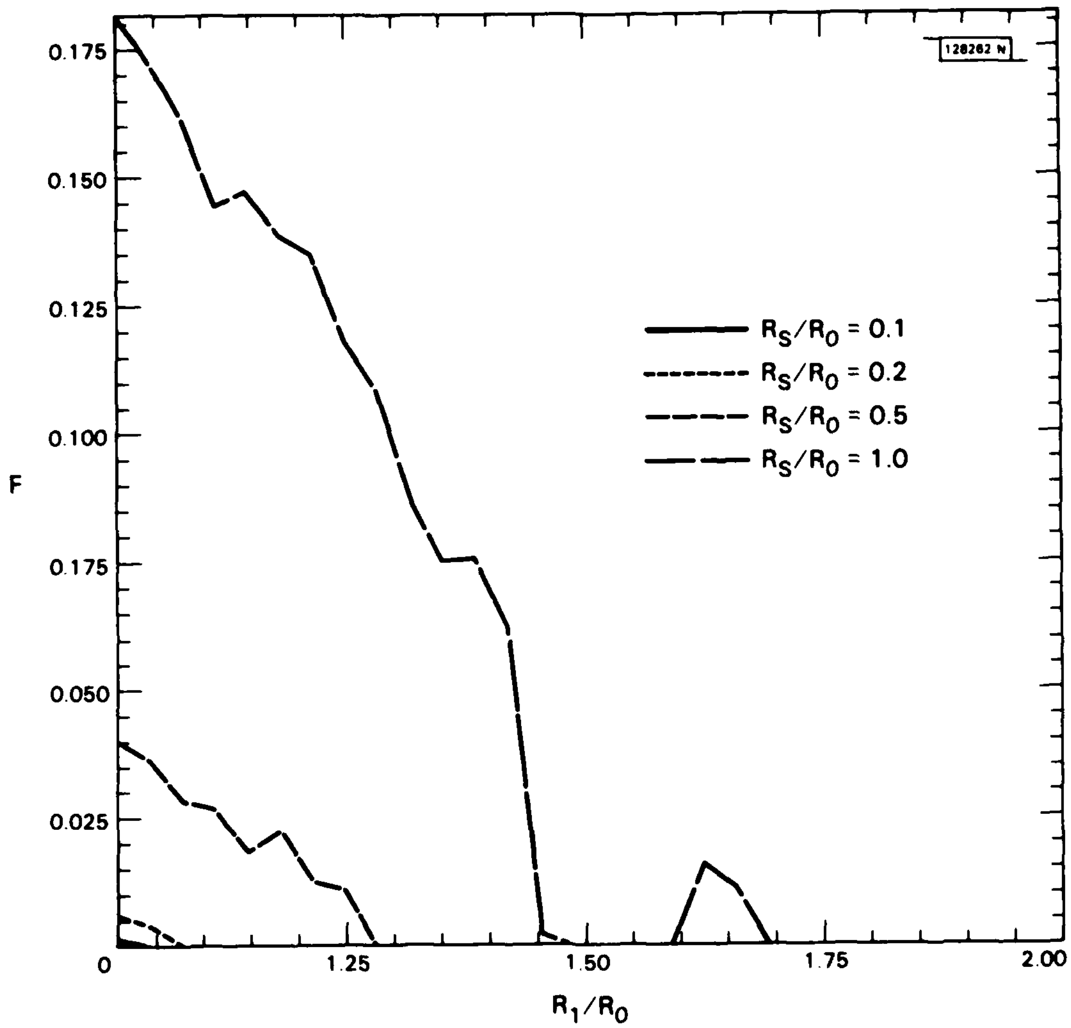

Fig. 4. Fraction of time two satellites with relative inclination $I = \pi/2$ are within the normalized range R_s/R_0 as a function of the ratio R_1/R_0 of the satellite orbital radii, for worse case angle η.

$$\alpha = \cos^2 \frac{I}{2} \sin^2 \frac{\omega_\Delta T_1}{2} \qquad \text{(IV-4)}$$

$$\alpha = \sin^2 \frac{I}{2} \sin^2 \frac{\omega_\Sigma T_2}{2} \qquad \text{(IV-5)}$$

valid for $\alpha < \cos^2 \frac{I}{2}$ *and* $\alpha < \sin^2 \frac{I}{2}$, *respectively.* T_1 *is the time duration* over which $\cos^2 \frac{I}{2} \sin^2 \frac{\omega_\Delta \tau}{2}$ rises from zero to α, while T_2 is the time duration over which $\sin^2 \frac{I}{2} \cos^2 \frac{\omega_\Sigma \tau + \eta}{2}$ rises from zero to α. Then fT_Δ, the time within T_Δ that (IV-2) is satisfied, is upper bounded by

$$fT_\Delta < (2T_1,\ 2 \frac{\omega_\Sigma}{\omega_\Delta} T_2) \quad ,^* \qquad \text{(IV-6)}$$

where the factor $\omega_\Sigma/\omega_\Delta$ in the second inequality reflects the number of zeros in an interval of length T_Δ. Using the series expansion of sin X to upper bound T_1 and T_2 as given in (IV-4) and (IV-5), and substituting into (IV-6),we obtain:

$$f < \left(\frac{2}{\pi} \frac{\alpha^{1/2}}{\cos I/2} \quad , \quad \frac{2}{\pi} \frac{\alpha^{1/2}}{\sin I/2} \right) \quad , \qquad \text{(IV-7)}$$

valid for all I.

A much tighter bound is easily derived for the case $\alpha \ll \cos^2 \frac{I}{2}$, $\alpha \ll \sin^2 \frac{I}{2}$, by counting only the contribution of the zeros in the $\sin^2 \frac{I}{2} \cos^2 \frac{\omega_\Sigma \tau + \eta}{2}$ term which occur in the region where the $\cos^2 \frac{I}{2} \sin^2 \frac{\omega_\Delta \tau}{2}$ term is small. Then,

---

* The notation A < (B,C) denotes A < B and A < C.

$$fT_\Delta < 2\,\frac{\omega_\Sigma}{\omega_\Delta}\,T_2 \cdot \frac{2T_1}{T_\Delta} \quad . \tag{IV-8}$$

After substitution, we have:

$$f < \frac{8}{\pi^2}\,\frac{\alpha}{\sin I} \quad , \tag{IV-9}$$

valid only for $\alpha \ll \cos^2 \frac{I}{2}, \sin^2 \frac{I}{2}$. This bound should be quite tight in the region of small α and I close to $\pi/2$. See Fig. 5.

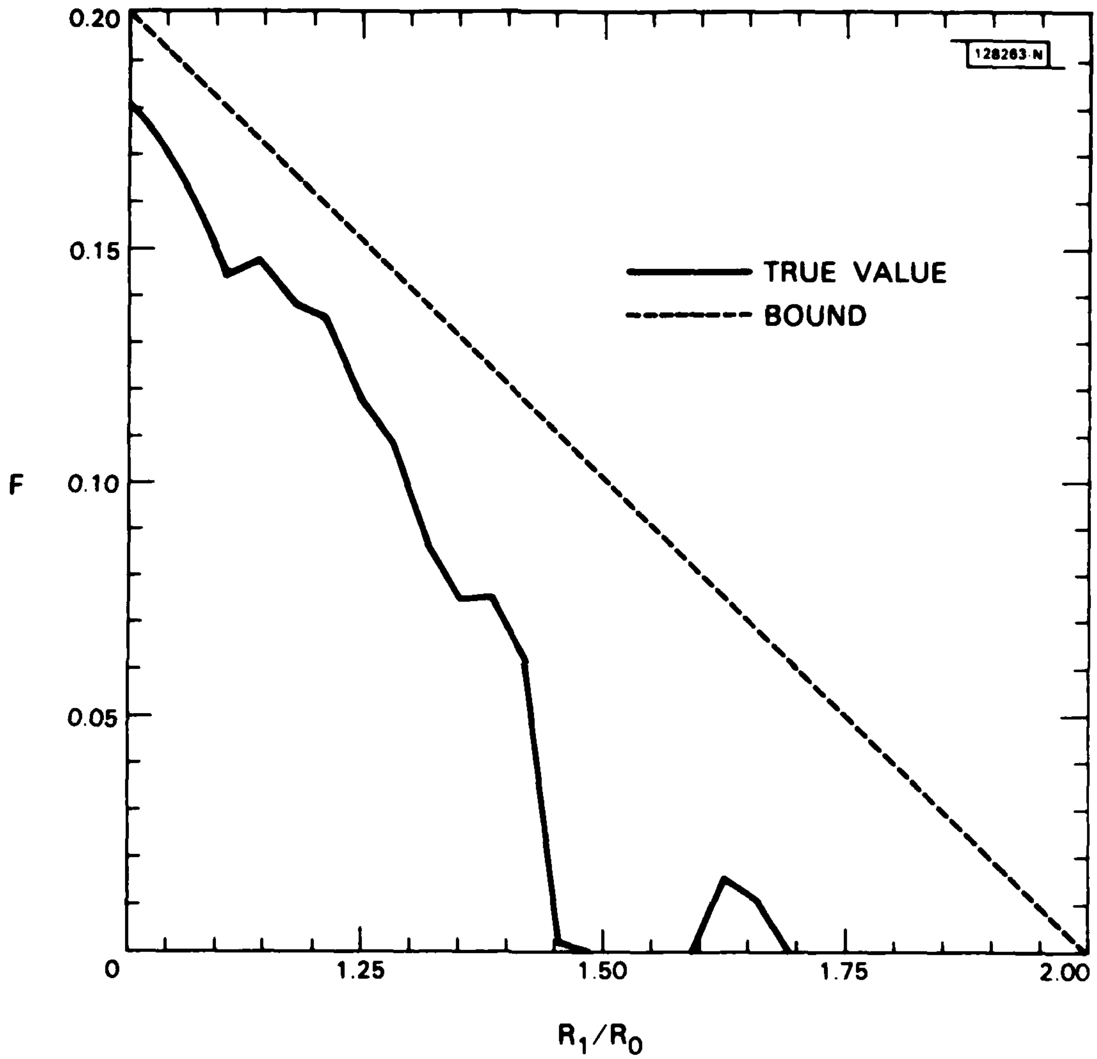

Fig. 5. Fraction of time two satellites are within normalized range R_s/R_o = 1.0, J = $\pi/2$.

V. APPLICATION TO SATELLITE-BORNE SEARCH SYSTEMS

Consider a search satellite with circular orbit radius R_0, and a sensor with a detection range R_s which scans at random within a spherical cap whose center lies on the earth-satellite line from satellite 0. The boundary of the cap is determined by the intersection of the sphere of radius R_1 ($R_1 > R_0$) which characterizes the satellite being searched for with a sphere of radius R_s centered on the search satellite. By symmetry the following analysis also applies for a search system with the searcher at altitude R_1 searching for satellites at altitude R_0, with a search system range of R_s. From the previous section it is clear that a satellite being searched for will lie within the cap a fraction f of the total time T_Δ. In this section we will relate the solid angle search rate of the sensor system on board the searching spacecraft, to the total aggregate time required to achieve the specified probability of detection against the intended targets.

The sensor system is characterized by the angular resolution cell area θ^2, the characteristic integration time τ, and the single look probability of detection p_d which occurs when the intended victim satellite is within one or more resolution cells at range no greater than R_s. N is the number of parallel detectors operating at the same time.

Within the characteristic time T_Δ, there will be fT_Δ/τ independent opportunities for the sensor to detect the satellite being searched for. Within the total time T_Δ, the aggregate cumulative probability of missing the target is given by

$$p_m \leq \left(1 - \frac{N\theta^2}{\pi\psi^2} p_d\right)^{fT_\Delta/\tau} \qquad \text{(V-1)}$$

Thus for a total search time T_s substantially greater than T_Δ we obtain

$$p_m \leq \left(1 - \frac{N\theta^2}{\pi\psi^2} p_d\right)^{(fT_\Delta/\tau)(T_s/T_\Delta)} \qquad \text{(V-2)}$$

In both of these expressions, the angular area of the cap is given by $\pi\psi^2$ steradians, where ψ is the included half angle given by $\cos\psi = \frac{R_0^{\ 2} + R_1^{\ 2} - R_s^{\ 2}}{2R_0R_1}$. It is easy to show, using (IV-3), that $\cos\psi = 1 - 2\alpha$. Hence for small α, $\pi\psi^2 \simeq 4\pi\alpha$.

For the case where the detector tests only a small part of the sky at a time, $N\theta^2/\pi\psi^2$ is small. Use of the exponential approximation yields

$$p_m \simeq e^{\frac{-fT_s}{\tau} \frac{N\theta^2}{4\pi\alpha} p_d} \tag{V-3}$$

Solving to obtain the expression for search time:

$$T_s \simeq \frac{4\pi\alpha \, \ln \, (1/p_m)}{fN\dot{\Omega} \, p_d} \tag{V-4}$$

where we have used the characteristic solid angle search rate:

$$\dot{\Omega} = \theta^2/\tau \tag{V-5}$$

In practice, the probability of detection for a sensor varies as a very strong power of the range to the target. In effect, there will be a characteristic range inside which the probability of detection will be essentially unity. Outside this range the probability of detection will be virtually zero. Therefore, for engineering purposes, we may approximate:

$$T_s \simeq \frac{4\pi\alpha \, \ln \, (1/p_m)}{f \, N \, \dot{\Omega}} \quad , \tag{V-6}$$

where f is computed on the basis of the characteristic range R_s corresponding to a detection probability p_d, say, .9.

On the other hand, if $N\theta^2 \geq \pi\psi^2$, the sensor continuously monitors the cap. Then

$$p_m = (1 - p_d)^{fT_s/\tau} \tag{V-7}$$

which gives

$$T_S = \frac{\tau}{f} \frac{\ln p_m}{\ln(1 - p_d)} \tag{V-8}$$

with the restriction $T_S \geq T_\Delta$. Generally such a system would be operated to achieve $T_S = T_\Delta$ with the given sensor characteristics.

VI. RANGE RATE

In this section the range rate is derived. Equation (III-4) may be differentiated to give:

$$\dot{d}(\tau) = \frac{R_0R_1}{d(\tau)} \left[\omega_\Delta \sin \omega_\Delta\tau \cos^2 \frac{I}{2} - \omega_\Sigma \sin (\omega_\Sigma\tau + \eta) \sin^2 \frac{I}{2}\right] \qquad \text{(VI-1)}$$

Since $|d(\tau)| \geq R_1 - R_0$, we may upper bound the maximum range rate as

$$\dot{d}(\tau) \leq \frac{R_0R_1}{R_0-R_1} \left[\omega_\Delta \cos^2 \frac{I}{2} + \omega_\Sigma \sin^2 \frac{I}{2}\right] \qquad \text{(VI-2)}$$

An equivalent form is

$$\dot{d}(\tau) < \frac{R_0R_1}{R_0-R_1} \left(\frac{2\pi}{T_0} - \frac{2\pi}{T_1} \cos I\right) \qquad \text{(VI-3)}$$

As it should, this expression illustrates the minimum peak range rate for coplanar satellites moving in the same direction ($I = 0°$), and maximum peak range rate for coplanar satellites moving in the opposite direction ($I = 180°$).

VII. ACKNOWLEDGMENT

L. N. Weiner cheerfully generated all the numerical and graphical data in this note.

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